Chapter 5 Linear Relations

Get Ready, pages 156–157
1. a) $1.30/l b) 18.83 m/s c) 625 students/year
d) $29/h e) 0.4 laps/min f) 2.5 min/lap
2. a) $17.56/h b) $316.08
3. a) Begin with 1 and continuously add 2; 9, 11
b) Begin with 12 and continuously subtract 5; –3, –8
c) Begin with 3 circles forming a triangle. Add 3 circles to the
bottom. Then add 4 circles to the bottom. Continue adding an
additional circle to the extra each time;

d) Begin with 4 small squares forming a large square. Add three
squares to one corner. Continue adding 3 squares to the next
corner;

4. Answers will vary. For example: –7, –4, –1, ...; answer: begin with –7 and continuously add 3; 2, 5
5. a) 0 b) 8
6. a) \( y = 15 \) b) \( y = 6 \)
7. a) \[
\begin{array}{c|c|c}
\text{Height (cm)} & \text{Area (cm}^2) & \text{Pattern Related to Height} \\
\hline
1 & 15 & 18 – 3(1) \\
2 & 12 & 18 – 3(2) \\
3 & 9 & 18 – 3(3) \\
4 & 6 & 18 – 3(4) \\
5 & 3 & 18 – 3(5) \\
\end{array}
\]
b) The points line up in a straight line.
8. D(2, 3), E(7, 0), F(–5, 0), G(–3, –1), H(2, –2)

5.1 Representing Patterns, pages 158–165

1. a) 

<table>
<thead>
<tr>
<th>Height (cm)</th>
<th>Area (cm)</th>
<th>Pattern Related to Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>18 – 3(1)</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>18 – 3(2)</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>18 – 3(3)</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>18 – 3(4)</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>18 – 3(5)</td>
</tr>
</tbody>
</table>

b) Begin with an area of 18 cm\(^2\) and decrease by 3 cm\(^2\) for each
increase of 1 cm in height.
c) \( A = 18 – 3h \)
2. a) 

<table>
<thead>
<tr>
<th>Depth (cm)</th>
<th>Wet Area (cm)</th>
<th>Pattern Related to Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>12(0)</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td>12(1)</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>12(2)</td>
</tr>
<tr>
<td>3</td>
<td>36</td>
<td>12(3)</td>
</tr>
<tr>
<td>4</td>
<td>48</td>
<td>12(4)</td>
</tr>
</tbody>
</table>

b) \( A = 12d \)
c) 100.8 cm\(^2\)
d) The dry area is the difference between the depth that is wet
multiplied by 12 and subtracted from 216 cm\(^2\).
e) \( A = 216 – 12d \)
f) 115.2 cm\(^2\)
3. a) 

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Depth (cm)</th>
<th>Pattern Related to Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>60</td>
<td>60 – 8(0)</td>
</tr>
<tr>
<td>1</td>
<td>52</td>
<td>60 – 8(1)</td>
</tr>
<tr>
<td>2</td>
<td>44</td>
<td>60 – 8(2)</td>
</tr>
<tr>
<td>3</td>
<td>36</td>
<td>60 – 8(3)</td>
</tr>
<tr>
<td>4</td>
<td>28</td>
<td>60 – 8(4)</td>
</tr>
</tbody>
</table>

b) Begin with 60 cm and subtract 8 cm multiplied by the time
in minutes.
c) \( d = 60 – 8t \) d) 24 cm
4. a) 

<table>
<thead>
<tr>
<th>Number of Days</th>
<th>Height of Plant (cm)</th>
<th>Pattern Related to Number of Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
<td>6 + 3(0)</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>6 + 3(1)</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>6 + 3(2)</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>6 + 3(3)</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
<td>6 + 3(4)</td>
</tr>
<tr>
<td>5</td>
<td>21</td>
<td>6 + 3(5)</td>
</tr>
</tbody>
</table>

b) The initial height of the plant was 6 cm. Add 3 cm times the
number of days after it was planted. c) \( h = 6 + 3t \) d) 96 cm
5. a) \( C = 1000 + 15.28n \)
b) The constant term is the $1000 initial cost. The numerical
coefficient is the $15.28 per book.
c) Discrete. The number of books is a whole number.
d) $14 752
6. a) Constant term: 150; numerical coefficient: –8; continuous,
because the height and time are measured allowing continuous change
b) Constant term: 60; numerical coefficient: 6; continuous,
because the depth of snow and time are measured allowing continuous change
c) Constant term: 40; numerical coefficient: 10; discrete, because
the software titles are counted using whole numbers
d) Constant term: 750 000; numerical coefficient: 500; discrete,
because the number of cows is counted using whole numbers
7. He did not divide by 2 to get the rate of change. The
numerical coefficient should be 5 instead of 10.
He used 100 as his starting point, but at time 0 s, the distance
would have been 90 m. The equation should be \( d = 90 + 5t \).
5.2 Interpreting Graphs, pages 166–177

1. Answers will vary. For example: a) 1170 km/h  b) 1192 km/h  c) 1236 km/h  d) 1104 km/h
2. a) 14 km/h; interpolation  b) 7 h
3. a) 14  b) 1.5
4. a) 30 m  b) 11 min
5. a)

b) Curve; this is a non-linear relation.

6. a)

b) The base time is 20 min, increasing by 5 min for every additional 100 m² of lawn area.
c) This is a linear relation because the rate of change from the chart is constant and the graph is a straight line.
d) Yes. The rate does not change below 100 m² or above 700 m².

7. a)

b) 31 km  c) 3.5 h
8. a)

Yes. The points form a straight line. The rate of change is a constant 5.5 m/s.
9. a) Answers will vary. For example: 200 bpm at age 20; 190 bpm at age 30; 150 bpm at age 70
b) Begin at 200 bpm at age 20 and decrease by 1 bpm per year.
c) The points form a straight line, and the rate of change is a constant 1 bpm/year.
d) No. Young people are still growing, so would have a different relationship. Seniors may also have a different relationship between maximum heart rate and age.

10. a) 

The length of the ladder belongs on the horizontal axis because the distance from the wall depends on the length.
b) approximately 58 cm
c) approximately 1.7 m
d) 118 cm

11. The initial cost is $4.00. The additional cost is $0.75/tonne.

12. a) Linear; both sets of points line up in a straight line, and the rates of change are both constant.
b) Blue ball; its bounce heights are greater.
c) Yes; the bounciness does not change and the relationship is linear.

13. a) Discrete; the bounces are counted using whole numbers.

14. Answers will vary. For example:
a) the cost of renting a boat for an afternoon
b) the hourly charge for renting a boat for a day

16. a)
b) Answers will vary. For example: There are three linear relations. The initial mass is 180 kg. The growth rate is greatest between ages 24 months and 36 months and least during the first 24 months.

17. a) The parachutist began the descent at an altitude of 2500 m and was accelerating until she opened her chute at a height of 1000 m. She then fell at a steady rate for 180 s, or 3 min, which is when she landed.
   b) After 20 s, the graph becomes a straight line, so she stopped accelerating.
   c) approximately 5.6 m/s

18. a) By subtracting consecutive values of the speed, the rates of change are all different. This is not a linear relation.
    b) By subtracting consecutive values of the speed, and dividing by the differences in the x-values, the rates of change are all equal. This is a linear relation.

19. In a table of values, a) and d) give constant rates of change, but b) and c) do not.

5.3 Graphing Linear Relations, pages 178–189

1. a)

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>Charge ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>60</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>140</td>
</tr>
<tr>
<td>3</td>
<td>180</td>
</tr>
<tr>
<td>4</td>
<td>220</td>
</tr>
</tbody>
</table>

2. a) Fixed: 20 °C; variable: 12 °C/min
    b) Fixed: $7.50; variable: $6.00/h
    c) Fixed: 0 s; variable: 21.4 s/MB

3. a) Initial volume of 150, so the vertical intercept is 150. The graph is declining by 8 L/min, which is the rate of change.

b) Volume of Water Remaining

4. a)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-11</td>
</tr>
<tr>
<td>-1</td>
<td>-8.5</td>
</tr>
<tr>
<td>0</td>
<td>-6</td>
</tr>
<tr>
<td>1</td>
<td>-3.5</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
</tbody>
</table>
5. a) Fixed term: 7; rate of change: 2
   b) Fixed term: −2; rate of change: −3
   c) Fixed term: 1.2; rate of change: 3.4
   d) Fixed term: −6; rate of change: −5
   e) Fixed term: 2; rate of change: $\frac{1}{3}$
   f) Fixed term: $\frac{1}{2}$; rate of change: $−\frac{1}{4}$

6. Answers will vary. For example: Create a table of values to find points on the line represented by the equation, draw the x- and y-axes, and plot the graph.

7. Horizontal, because $y$ has a constant value and $x$ can be any number.
9. a) Vertical, because the input is constant and the outcome varies.
   b) Horizontal, because the input varies and the outcome is constant.
10. a) D   b) A   c) B   d) C
11. All have the same rate of change, so the lines are parallel.
The y-intercepts are all different. They are
   a) 5   b) -8   c) 0   d) -1
12. All have the same y-intercept. The rates of change are all different. They are
   a) 8   b) 0   c) -1   d) 2
13. b) As \( a \) increases, the graph becomes steeper; as \( a \) decreases, the graph becomes less steep.
c) As \( b \) increases, the graph shifts upward; as \( b \) decreases, the graph shifts downward.
d) Answers will vary. For example: \( a \) represents the rate of change or steepness of the graph, and \( b \) represents the y-intercept.
14. They both begin at the point (0, 0), so they have the same y-intercept.
15. The fixed cost is $20. The variable cost is $0.15 per minute spent calling long distance.

<table>
<thead>
<tr>
<th>Long Distance (min)</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>21.5</td>
</tr>
<tr>
<td>20</td>
<td>23</td>
</tr>
<tr>
<td>30</td>
<td>24.5</td>
</tr>
<tr>
<td>40</td>
<td>26</td>
</tr>
<tr>
<td>50</td>
<td>27.5</td>
</tr>
</tbody>
</table>

16. Answers will vary. For example:
a) printing logos on shirts to sell at a concert
b) an hourly employee's earnings
17. a) 

b) As the altitude increases, the temperature decreases by 6.5 °C/km.

18. a) 

[Graphs showing height and bone length for women and men]
b) For all three types of bones, men begin at a smaller height relative to the bone length, but that increases as the length of the bone increases.
c) approximately 159 cm
d) approximately 176 cm
e) femur: female: 45.6 cm; male: 44.6 cm;
tibia: female: 36.7 cm; male: 35.9 cm;
humerus: female: 33.3 cm; male: 31.0 cm
f) femur
20. a) 150 mg
   b) 192 mg

Race: Tommy vs Harold

b) 175 m
c) Harold by 27 m/min
d) Harold; approximately 6.7 min
e) After about 5.5 min
22. ground distance: \( d = 69.12t \); climbing distance: \( d = 20.16t \)

5.4 Equations of Linear Relations, pages 190–199

1. a) The \( y \)-intercept occurs where \( x = 0 \). Enter the values of \( x \) and \( y \) in separate columns, with \( x \) increasing by a constant amount. Add a third column to show how \( y \) is changing. Determine the rate of change by dividing the change in \( y \) by the change in \( x \).
b) The \( y \)-intercept occurs where \( x = 0 \). Use the ordered pairs for two points. Subtract the \( y \)-values. Subtract the \( x \)-values. Divide the difference in the \( y \)-values by the difference in the \( x \)-values to determine the rate of change.

2. a) $80; it is the vertical intercept
   b) The cost increases by $40 per hour.
c) \( C = 80 + 40t \)

3. a)

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Energy Burned (kcal)</th>
<th>Pattern per 2 min</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>19.6</td>
<td>19.6</td>
</tr>
<tr>
<td>4</td>
<td>39.2</td>
<td>19.6</td>
</tr>
<tr>
<td>6</td>
<td>58.8</td>
<td>19.6</td>
</tr>
<tr>
<td>8</td>
<td>78.4</td>
<td>19.6</td>
</tr>
<tr>
<td>10</td>
<td>98.0</td>
<td>19.6</td>
</tr>
</tbody>
</table>

b) There is no initial energy burned.
c) Energy is burned at a rate of 9.8 kcal/min.
d) \( E = 9.8t \)  e) 294 kcal

4. It would pass through the origin \((0,0)\).
5. a) C  b) B  c) A

6. a)

<table>
<thead>
<tr>
<th>( x )</th>
<th>A</th>
<th>Pattern Between Terms</th>
<th>B</th>
<th>Pattern Between Terms</th>
<th>C</th>
<th>Pattern Between Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>11</td>
<td>5</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>12</td>
<td>-3</td>
<td>5</td>
<td>0</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>-3</td>
<td>5</td>
<td>0</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>-3</td>
<td>5</td>
<td>0</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>-3</td>
<td>5</td>
<td>0</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

b) All three graphs have a \( y \)-intercept of 5, but their rates of change are different.
c) The constant term is 5 for each equation, but the numerical coefficient is different. For A, \( a = -3 \); for B, \( a = 0 \); for C, \( a = 2 \).

7. a) Shania. Her distance-intercept is smaller than Dania's.
   b) Shania. Her line is steeper.
c) \( d = 100 - 7t \)

8. a) 9.5 L/100 km
   b) \( F = 0.095d \)
   c) The numerical coefficient is the fuel efficiency divided by 100.
   d) \( F = 0.07125d \)

9. a) \( y = 0.5x - 1 \)  b) \( x = 4 \)

11. a) \( y = 2x + 1 \)  b) \( y = 2x - 6 \)

13. a) No, his average speed is less than 50 km/h.
   b) No, his average speed is less than 40 km/h.
   c) Yes, his average speed is greater than 30 km/h.
14. A: y = 3; B: x = 2; C: y = -3; D: x = -2
15. a) Lacrosse: 2.8 kJ/min; swimming: 1.6 kJ/min; cycling: 1.2 kJ/min; walking: 0.8 kJ/min
   b) Lacrosse: E = 2.8t; swimming: E = 1.6t; cycling: E = 1.2t; walking: E = 0.8t
16. a) L = 0.23m + 25
   b) The constant term of 25 represents the length of the spring with no weight on it; the variable term of 0.23 represents the change in length of 0.23 cm for each additional gram of mass on the weight.
   c) 94 cm
   d) approximately 21.7 g
17. a)
   b) The points form a straight line and the rate of change is 1, so the relation is linear.
   c) y = -x + 8
18. a)
   b) Answers will vary. For example: (1, -2)
   c) y = -3x + 1
   d) y = -3(1) + 1 = -2; The point satisfies the equation.
19. a) y = -3x + 6
20. a)
   b) Gavin drove at a constant speed.
   c) 108 km
   d) approximately 2.2 h
   e) d = 90t
   f) 90 km/h

Rich Problems, page 200
1. a) i) The initial value of the collection was $215 and it increased by $100 per year.
   ii) The initial value of the collection was $2000 and it decreased by $5 per year.
   iii) The initial value of the collection was $0 and it increased by $120 per year.